Delhi Public School Guwahati
"Under the aegis of the Delhi Public School Society, Delhi"

Holiday homework,Class XII,2016-17

Ch-Continuity and Differentiability

1. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$.

2. If $y = x\log\left(\frac{x}{a+bx}\right)$, then prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

3. If $y = \log\left(x + \sqrt{x^2 + a^2}\right)$, then $\left(x^2 + a^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

4. If $x = a\sin t$ and $y = a\left(\cos t + \log\frac{t}{2}\right)$, then find $\frac{d^2y}{dx^2}$.

5. Differentiate the following w.r.t x: $\sin^{-1}\left[\frac{2^{x+1}.3^x}{1+(36)^x}\right]$.

6. If $x\sin(a+y) + \sin a\cos(a+y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

7. If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

8. If $y^x = e^{y-x}$,then prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.

9. If $\sin y = x\sin(a+y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

10. If $y = x^{\sin x - \cos x} + \frac{x^2-1}{x^2+1}$, then find $\frac{dy}{dx}$.

11. If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

12. If $x = \tan\left(\frac{1}{a}\log y\right)$, then show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$

13. If $y = a\sin x + b\cos x$, then prove that $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$.

14. If $y = \sqrt{x^2+1} - \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$, then find $\frac{dy}{dx}$.

15. If $y = \cos^{-1}\left[\frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}}\right]$, then find $\frac{dy}{dx}$

16. If $y = \cos^{-1}\left[\frac{3x + 4\sqrt{1-x^2}}{5}\right]$, then find $\frac{dy}{dx}$

17. If $y=\sin^{-1}\left[\frac{5x+12\sqrt{1-x^2}}{13}\right]$, then find $\frac{dy}{dx}$

Ch- Inverse Trigonometric Functions

18. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$

19. Prove that $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right]=\sqrt{\frac{1+x^2}{2+x^2}}$.

20. Prove that $\sin\left[\cot^{-1}\left\{\cos\left(\tan^{-1}x\right)\right\}\right]=\sqrt{\frac{1+x^2}{2+x^2}}$.

21. Prove that $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$.

22. Prove that $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{8}{19}\right)=\frac{\pi}{4}$.

23. Write the principal values of the following:

a) $\left[\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\left(-\frac{1}{2}\right)\right]$

b) $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

c) $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$

d) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

e) $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

24. Prove that $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$

25. Prove that $\cos^{-1}\left(\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right)=2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$.

26. Prove that $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=1, \quad 0<x<1$

27. If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$, prove that $\sin y=\tan^2\frac{x}{2}$.

28. Show that $\cos\left(2\tan^{-1}\frac{1}{7}\right)=\sin\left(4\tan^{-1}\frac{1}{3}\right)$.

29. Solve : $3\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+2\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$.

30. Prove that: $\tan^{-1}2+\tan^{-1}3=\frac{3\pi}{4}$

31. Prove that: $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$.

32. Prove that: $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$.

33. Prove the : $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

34. Prove that: $\cos^{-1}\left(\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right)=2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$.

35. Prove that: $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$

36. Solve for x: $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$, $-1<x<1$.

37. Write the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

38. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, prove that $x^2+y^2+z^2+2xyz=1$.

Ch-Applications of Derivatives

39. Find the intervals in which the function $f(x)=\sin x-\cos x$, $0\le x\le 2\pi$ is strictly increasing or strictly decreasing. (Ans: $\left(0,\frac{3\pi}{4}\right),\left(\frac{3\pi}{4},\frac{7\pi}{4}\right),\left(\frac{7\pi}{4},2\pi\right)$)

40. Find the intervals in which the function $f(x)=\sin x+\cos x$, $0\le x\le 2\pi$ is increasing or decreasing.

(Ans: $\left[0,\frac{\pi}{4}\right],\left[\frac{\pi}{4},\frac{5\pi}{4}\right],\left[\frac{5\pi}{4},2\pi\right]$)

41. Find the intervals in which the function $f(x)=\sin x+\cos x$, $0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

42. Find the intervals in which the function $f(x)=20-9x+6x^2-x^3$ is strictly increasing or strictly decreasing. (Ans: $(-\infty,1),(1,3),(3,\infty)$)

43. If $f(x)=3x^2+15x+5$, then find the approximate value of $f(3.02)$, using differentials.

(Ans: 77.66)

44. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (Ans: 2ab)

45. An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$.

46. Show that semi-vertical angle of the cone of maximum volume and of given slant height is $\cos^{-1}\frac{1}{\sqrt{3}}$.

47. Find the equation of tangents to the curve $y=\cos(x+y)$, $-2\pi\le x\le 2\pi$ that are parallel to the line $x+2y=0$.

48. Using differentials find the approximate value of $\tan 46°$ if it is being given that $1°=0.01745$.

49. It is given that for the function $f(x)=x^3+bx^2+ax$, $x\in[1,3]$, Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$. Find a and b.

Ch-Relations and Functions

50. Let the function $f:R\to R$ be defined by $f(x)=\cos x$, $x\in R$. Show that f is neither one-one nor onto.

51. Functions $f,g:R\to R$ are defined respectively, by $f(x)=x^2+3x+1$, $g(x)=2x-3$, find

a) fog b) gof c) fof d) gog

52. If R_1 and R_2 are two equivalence relations in a given set A, show that $R_1 \cap R_2$ is also an equivalence relation.

53. Let Z be the set of all integers and R be the relation on Zdefined as $R = \{(a,b) : a,b \in Z \text{ and } (a-b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.

54. Find the number of equivalence relations on the set $\{a,b,c\}$ containing (a,b) and (b,a).

Ch- Matrices and Deteminants

55. If $a_1, a_2, a_3 ....a_r$ are in G.P, then prove that the determinant $\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix}$ is independent of r.

56. Using properties of determinants find the value of the determinant $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$.(Ans:0)

57. If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = A(A+I)$, where I is identity matrix of order 3.