SUMMER HOLIDAY ASSIGNMENT

MATHEMATICS (CLASS : XI)

CHAPTER 1 : SETS

1. Which of the following are sets? Justify your answers:
   (i) The collection of difficult topics in Mathematics.
   (ii) The collection of all good students of class XI.
   (iii) The set of all prime numbers more than 10.
   (iv) The collection of excellent cars in the city.
   (v) The collection of the students of in a class scoring more than 90% in Science.

2. Describe the following sets by Roster method:
   (i) The set of the vowels of the word 'MADANAMOHANA'.
   (ii) The set of all states of India beginning with the letter A.
   (iii) The set of integers between $-\frac{9}{2}$ and $\frac{11}{3}$.
   (iv) The set of all natural numbers 'x' such that $4x + 9 < 50$.
   (v) The set of all integers 'x' such that $2x - 5 > 3$.
   (vi) {x:x is a two digit number such that the sum of digits is 6}
   (vii) {x:x is a consonant in the English alphabet which succeeds P}
   (viii) {x : x is a positive integer and $x^2 < 20$}
   (ix) {x: x is a positive integers and divisor of 21}
   (x) $\{a_n: n \in N, a_{n+1} = 4a_n \text{ and } a_1 = 4\}$

3. Describe the following sets by set builder method:
   (i) {1,2,3,4,5,6,7,8,9,10}
   (ii) {1,3,5,7,9}
   (iii) {0,3,6,9,12,…}
   (iv) {5,10,15,20,25}
   (v) {1,4,7,10,…}
   (vi) {2,6,10,14,…}
   (vii) {3,8,13,18,…}
   (viii) {4,7,10,13,…}
   (ix) {5,11,17,23,…}
   (x) {-1,1}
   (xi) {14,21,28,35,42,…,98}
   (xii) {53,59,61,67,71,73,79,83,89,97}
   (xiii) {1,8,27,64,125}
   (xiv) $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\}$
   (xv) $\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}\}$
   (xvi) $\{\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}\}$

4. Consider the following sets ϕ, A = {a,e}, B = {e, i}, C = {a, e, i, o, u} and fill in the blank with either $\subset$ or $\not\subset$ (i) ϕ.....B (ii) A.... B (iii) A.....C (iv)B.....C

5. Let A = { ϕ , { ϕ }, 1, {1, ϕ },7}; which of the following are false ? Justify .
   (i) $\phi \in A$ (ii) $\{\phi\} \in A$ (iii) $\{1\} \in A$ (iv) $\{7, \phi\} \subset A$ (v) $7 \subset A$
   (vi) $\{7, \{1\}\} \subset A$ (vii) $\{\{7\}, \{1\}\} \not\subset A$ (viii) $\{\{\phi\}\} \subset A$

6. Prove the following results :
   (i) $A - B = A \cap B'$.
   (ii) $A - B = A \Leftrightarrow A \cap B = \phi$.
   (iii) $(A - B) \cup B = A \cup B$.
   (iv) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

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7. If A, B and C are three non-empty finite sets such that n(A) = 19, n(B) = 15, n(C) = 17, n(A ∩ B) = 11, n(B ∩ C) = 6, n(C ∩ A) = 7 and n(A ∩ B ∩ C) = 5. Also n (U) = 50. Find (i) n(A ∩ $B^{/}$ ∩ $C^{/}$) (ii) n(B ∩ $C^{/}$ ∩ $A^{/}$) (iii) n(C ∩ $A^{/}$ ∩ $B^{/}$) (iv) n(A ∩ B ∩ $C^{/}$) (v) n(B ∩ C ∩ $A^{/}$) (vi) n(C ∩ A ∩ $B^{/}$) (vii) n(A ∪ B ∪ C)$^{/}$
   [Ans : (i) 6, (ii) 3, (iii) 9, (iv) 15, (v) 13, (vi) 17, (vii) 18]

8. A class has 175 students and number of students with Mathematics 100; Physics 70; Chemistry 46; Mathematics and Physics 30; Mathematics and Chemistry 28, Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. Find (i) the number of students who are enrolled in Mathematics alone, Physics alone and Chemistry alone; (ii) the number of students who have not taken any of these three subjects.
   [Ans: (i) Mathematics alone 60, Physics alone 35, Chemistry alone 13 (ii) 12]

9. In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all three subjects. Find the number of students that had taken : (i) only Chemistry (ii) only Mathematics (iii) only Physics (iv) Physics and Chemistry but not Mathematics (v) Mathematics and Physics but not Chemistry (vi) only one of the subjects (vii) at least one of the three subjects (viii) none of the three subjects.
   [Ans: (i) 5 (ii) 5 (iii) 3(iv) 1 (v) 5 (vi) 13(vii) 24 (viii) 1]

CHAPTER 2 : RELATIONS AND FUNCTIONS

1. If f(x) = x + $\frac{1}{x}$, prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$.
2. If f(x) = $\frac{1}{2x+1}$, x ≠ - $\frac{1}{2}$, then show that f(f(x)) = $\frac{2x+1}{2x+3}$, provided x ≠-$\frac{3}{2}$.
3. Find the domain and range of the following functions :

   (i) f(x) = $\frac{1}{x-3}$ (ii) f(x) = $\frac{1}{x^2-4}$ (iii) f(x) = $\frac{1}{9-x^2}$ (iv) f(x) = $\frac{1}{\sqrt{7-x}}$

   (v) f(x) = $\frac{3}{2-x^2}$ (vi) f(x) = $\frac{x^2-4}{x-2}$ (vii) f(x) = $\frac{x-1}{1-x}$ (viii) f(x) = 1 - |x – 2|

   (ix) f(x) = $\frac{1}{\sqrt{x-[x]}}$ (x) f(x) = $\frac{|x-2|}{x-2}$ (xi) f(x) = x – [x] (xii) f(x) = $\sqrt{16-x^2}$

   (xiii) f(x) = $\sqrt{x^2-9}$ (xiv) f(x) = $\frac{x}{1+x^2}$ (xv) f(x) = $\frac{x^2}{1+x^2}$

   [Ans : (i) R – {3}; R – {0} (ii) R – {-2, 2} ; (-∞, - $\frac{1}{4}$]∪(0,∞) (iii) R - {-3, 3}; (-∞,0) ∪ [$\frac{1}{9}$, ∞)

   (iv) (-∞,7); (0, ∞) (v) R – {- $\sqrt{2}$, $\sqrt{2}$ }; (-∞, 0]∪($\frac{3}{2}$,∞) (vi) R – {2}; R – {4} (vii) R – {1}; {-1}

   (viii) R; (-∞, 1] (ix) R – Z; (0, ∞) (x) R – {2}; {-1,0,1} (xi) R, [0,1) (xii) [-4,4]; [0,4] (xiii) (-∞, 3]∪[3, ∞); [0, ∞) (xiv) R; [-½ , ½] (xv) R; [0,1)]

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CHAPTER 3 : TRIGONOMETRIC FUNCTIONS

1. Find the angle (in degree) subtended at the centre of the circle of radius 100 cm of an arc length 200 cm. [Ans. $114^0\ 32'\ 43''$]

2. The angles of a triangle are in A.P. The greatest angle is 5 times the least. Find the angles in circular measure. [Ans. $\pi/9, \pi/3, 5\pi/9$]

3. A horse is tied to a post by a rope 30 m long. If the horse moves along the circumference of the circle always keeping the rope tight, find how far it has gone when the rope has traced an angle 105^0. [Ans. 55 m]

4. The minute hand of a clock is 1.5 cm long. How far does the tip of the hand move in 50 minutes? [Ans. 7.86 cm]

5. A train is travelling on a circular path of 500 m radius at the rate of 40 km/h. Through what angle (in degree) has it moved in 10 seconds? [Ans. $1^0\ 16'\ 22''$]

6. Find sin (A + B), sin (A – B), cos (A + B), sin (A – B), tan(A + B) and tan (A – B) if
   (i) Cos A = 12/13, sin B = 4/5; $\pi/2 < A < \pi$, $0 < B < /2$
7. Prove that :
   (i) $\text{Cos}\left(\frac{3\pi}{4}+x\right)\text{-cos}\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\ \sin x$
   (ii) $\cos 510^0 \cos 330^0 + \sin 390^0 \cos 120^0 = -1$
   (iii) $\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}=2.$
   (iv) $\tan 720^0 - \cos 270^0 - \sin 150^0 \cos 120^0 = \frac{1}{4}$.
   (v) $\sec\left(\frac{3\pi}{2}-\theta\right)\sec\left(\theta-\frac{5\pi}{2}\right)+\tan\left(\frac{5\pi}{2}+\theta\right)\tan\left(\theta-\frac{3\pi}{2}\right)=-1$
   (vi) $\sin\frac{7\pi}{12}\cos\frac{\pi}{4}-\cos\frac{7\pi}{12}\sin\frac{\pi}{4}=\frac{\sqrt{3}}{2}.$
   (vii) $\frac{\sin(A-B)}{\sin A \sin B}+\frac{\sin(B-C)}{\sin B \sin C}+\frac{\sin(C-A)}{\sin C \sin A}=0$.
8. If $A + B = \frac{\pi}{4}$, prove that (i) $(1 + \tan A)(1 + \tan B) = 2$ (ii) $(\cot A - 1)(\text{Cot } B - 1) = 2$.
9. Find the general solutions :
   (i) $\tan x = 2 - \sqrt{3}$ (ii) $\sin mx + \sin nx = 0$ (iii) $\sin x - \cos x = \sqrt{2}$
   (iv) $\sin x - \cos x = 1$ (v) $\sqrt{3}\sin x - \cos x = \sqrt{2}$ (vi) $\cos x + \tan 2x + \cos 3x = 0$
   (vii) $\tan x + \tan 2x + \tan x \tan 2x = 1$
   (viii) $\tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x$
   (ix) $2 \tan x - \cot x + 1 = 0$
   (x) $\tan x + \sec x = \sqrt{3}$
   (xi) $2 \cos 2x + 3 \sin x = 0$

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CHAPTER 4 : PRINCIPLE OF MATHEMATICAL INDUCTION

Using principle of Mathematical Induction prove the following ($n \in N$) :

1. $1 + 4 + 7 + \ldots + (3n - 2) = \frac{1}{2}n\,(3n - 1)$.
2. $1.3 + 2.4 + 3.5 + \ldots + n(n + 2) = \frac{1}{6}\,n\,(n + 1)\,(2n + 7)$.
3. $2^n < 3^n$.
4. $1^2 + 2^2 + 3^2 + \ldots + n^2 > \frac{n^3}{3}$.
5. $x^n - y^n$ is divisible by $(x - y)$.
6. $x^{2n} - 1$ is divisible by $(x - 1)$.
7. $3^{2n} - 1$ is divisible by 8.
8. $5^{2n} - 1$ is divisible by 24.
9. $2^{3n} - 1$ is divisible by 7.
10. $5^{n-1} - 1$ is divisible by 4.
11. $2.7^n + 3.5^n - 5$ is divisible by 24.
12. $12^n + 25^{n-1}$ is divisible by 13.
13. $9^n - 8n - 1$ is divisible by 64.
14. $n(n+1)(2n+1)$ is divisible by 6.
15. Sum of the cubes of three consecutive natural numbers is divisible by 9.

CHAPTER 5 : COMPLEX NUMBERS

1. Find the modulus of : (i) $-i$ (ii) $3 + \sqrt{-5}$ (iii) $(3i - 1)^2$
2. Write the conjugate of : (i) $-3 + \sqrt{-2}$ (ii) i^3 (iii) $(3 + 4i)^2$
3. Express in a + ib form : (i) $\frac{i}{(1+i)}$ (ii) $\frac{1}{(-1+\sqrt{3}i)}$ (iii) $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$
4. Show that $\left\{\frac{(\sqrt{7}+i\sqrt{3})}{(\sqrt{7}-i\sqrt{3})} + \frac{(\sqrt{7}-i\sqrt{3})}{(\sqrt{7}+i\sqrt{3})}\right\}$ is purely real.
5. Find the values of θ for which $\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right)$ is purely real.
6. Find the square root of the following complex numbers :
   (i) $-8 - 6i$ (ii) $5 - 12i$ (iii) $3 - 4\sqrt{7}i$ (iv) $4 + 6\sqrt{-5}$ (v) $-2 + 2\sqrt{3}i$.